17/03/25

Unit - IV

Vector Calculus

vector differentiation
1. Velocity, Acc, Angle betn tangent vectors
2. D.D
3. Irrotational, Solenoid

Vector Integration:-
1. Line Integral
2. Green's Theorem.
3. Stoke's Theorem
4. Guass Divergence theorem.

Vector differentiation:-
1. velocity $(v) = \frac{dr}{dt}$

$$\bar{a} = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

2. Angle betn tangent vectors T_1 & T_2 is given by,

$$\cos\theta = \frac{T_1 \cdot T_2}{|T_1| \cdot |T_2|}$$

3. a) Gradient = grad $\phi = \Delta\phi = \bar{i}\frac{d\phi}{dx} + \frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial z}$

b) Divergence = $Div\bar{F} = \nabla\cdot\bar{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$

c) Curl $\bar{f} = \nabla\times\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$

Imp.

4. D.D $= [\nabla\phi]_P \cdot \hat{u}$

where $\hat{u} = \frac{\bar{u}}{|\bar{u}|}$

Imp

5. a) Solenoidal $\therefore \nabla\bar{f} = 0$

b) Irrotational curl $\bar{f} = \nabla\times\bar{f} = \bar{0}$

$$\phi = \int_{y,z\,const} f_1\,dx + \int_{x\text{-}free,\ z=const} f_2\,dy + \int_{x,y\,free} f_3\,dz.$$

- Vector Integration

1. Line Integral:- (curve)

$$\int_C \bar{F}\cdot d\bar{r} = \int_C f_1dx + f_2dy + f_3dz$$

2. Green's theorem:- (surface)

$$\oint_C \bar{F}\cdot d\bar{r} = \oint_C u\,dx + v\,dy$$

$$= \iint_R \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy.$$

3. Stokes theorem:-

$$\int_C \bar{F}\cdot d\bar{r} = \iint_S (\nabla\times\bar{F})\cdot\hat{n}\,ds.$$

4. Guass- Divergence theorem.

$$\iint_S \bar{F}\cdot\hat{n}\,ds = \iiint_V \nabla\cdot\bar{F}\cdot dv$$

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Q1] find the directional derivative of

~~$\phi = x + xy^2 +$~~

$\phi = xy^2 + yz^2 + zx^2$ at the point $P(-1,-1,0)$ in the direction of line PQ where $Q(1,1,1)$

→ Given:-

$\phi = xy^2 + yz^2 + zx^2$

$P \equiv (-1,-1,0)$

$Q \equiv (1,1,1)$

Since $D.D = [\nabla\phi]_P \cdot \hat{u}$ where $\hat{u} = \frac{\bar{u}}{|\bar{u}|}$

Step-1:- To find $\nabla\phi$

$$\nabla\phi = \bar{i}\frac{d\phi}{\partial x} + \bar{j}\frac{\partial\phi}{\partial y} + \bar{k}\frac{\partial\phi}{\partial z}$$

$$= \bar{i}\frac{\partial}{\partial x}(xy^2+yz^2+zx^2) + \bar{j}\frac{\partial}{\partial y}(xy^2+yz^2+zx^2) + \bar{k}\frac{\partial}{\partial z}(xy^2+yz^2+zx^2)$$

$$\cancel{[\nabla\phi]_P = \bar{i}(2y+2z) + \bar{j}(2x+2z) + \bar{k}(2y+2x)}$$

$$[\nabla\phi]_P = \bar{i}(y^2+2xz) + \bar{j}(\cancel{x^2}+2xy) + \bar{k}\,\cancel{(y+2xz)}\,(2yz+x^2)$$

$$[\nabla\phi]_{P(-1,-1,0)} = \bar{i}[(-1)^2+2(0)(-1)] + \bar{j}[2(-1)(-1)+0^2] + \bar{k}[2(-1)(0)+(-1)^2]$$

$$= \bar{i}[1+(0)] + \bar{j}[2] + \bar{k}[0+1]$$

$$\boxed{[\nabla\phi]_{P(-1,-1,0)} = +\bar{i} + 2\bar{j} + 1\bar{k}}$$

$\hat{u}$, $P = (-1,-1,0)$ $Q = (1,1,1)$

vector along the line joining (x_1, y_1, z_1) (x_2, y_2, z_2)

$(x_2 - x_1)\bar{i} + (y_2 - y_1)\bar{j} + (z_2 - z_1)\bar{k} = \bar{u} = \overline{PQ}$

$(1-(-1))\bar{i} + (1-(-1))\bar{j} + (1-0)\bar{k} = \bar{u}$

$2\bar{i} + 2\bar{j} + \bar{k} + \bar{u}$.

Q2] show that the vector field, $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational. Also find ϕ such that $\bar{F} = \nabla\phi$.

→ Given:-

$\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$

comparing with,

$\bar{F} = F_1\bar{i} + F_2\bar{j} + F_3\bar{k}$

where $F_1 = (x^2 - yz)$, $F_2 = (y^2 - zx)$, $F_3 (z^2 - xy)$

Step1:- To find curl $\bar{F}$

$$\therefore \text{curl } \bar{F} = \nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 - yz & y^2 - zx & z^2 - xy \end{vmatrix}$$

$$= \bar{i}\left[\frac{\partial}{\partial y}(z^2 - xy) - \frac{\partial}{\partial z}(y^2 - zx)\right] - \bar{j}\left[\frac{\partial}{\partial x}(z^2 - xy) - \frac{\partial}{\partial z}(x^2 - yz)\right] + \bar{k}\left[\frac{\partial}{\partial x}(y^2 - zx) - \frac{\partial}{\partial y}(x^2 - yz)\right]$$

$$= \bar{i}[(-1) - (-1)] - \bar{j}[(-1) - (-1)] + \bar{k}\{[(-1) - (-1)]$$

$$= \bar{i}[-1+1] - \bar{j}[-1+1] + \bar{k}[-1+1]$$

$$= \bar{i}[0] - \bar{j}[0] + \bar{k}[0]$$

$$\boxed{\nabla \times \bar{F} = \bar{0}}$$ //

$\therefore \bar{F}$ is irrotational.

Step ii) To find scalar potential function ϕ such that $\bar{F} = \nabla \phi$.

$$\phi = \int_{} f_1 dx + \int_{x = \text{free}} f_2 dy + \int_{x, y \text{ free}} f_3 dz + c$$

$$\phi = \int (x^2 - yz)\, dx + \int (y^2 - zx)\, dy + \int (z^2 - xy)\, dz$$

$$\phi = \frac{x^3}{3} - xyz + \frac{y^3}{3} + \frac{z^3}{3} + c$$

$$\boxed{\phi = \frac{x^3}{3} - xyz + \frac{y^3}{3} + \frac{z^3}{3} + c}$$

Q3] Find the angle between tangent to the curve $x = t+1$, $y = t^2+1$, $z = t^3-1$ at $t = 1$ & $t = -1$.

→ Given:- $x = t+1$, $y = t^2+1$, $z = t^3-1$ at $t = 1$ & $t = -1$

position vector $= \bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$$\bar{r} = (t+1)\bar{i} + (t^2+1)\bar{j} + (t^3-1)\bar{k}$$

Tangent vector $= \boxed{T = \frac{d\bar{r}}{dt}}$

$$\bar{r} = \cancel{\phi}\,\bar{i} + (2t)\bar{j} + (3t^2)\bar{k}$$

At $t = 1$

$\bar{T_1} = [T_1]_{t=1} \quad 2\bar{j} + 3\bar{k}$

At $t = -1$

$\bar{T_2} = [T_2]_{t=-1} = -2\bar{j} + 3\bar{k}$

$$\cos\theta = \frac{\bar{T_1} \cdot \bar{T_2}}{|\bar{T_1}| \cdot |\bar{T_2}|} \quad \text{--- (1)}$$

$$= \frac{(2\bar{j} + 3\bar{k}) \cdot (-2\bar{j} + 3\bar{k})}{\sqrt{\cancel{(2)^2}}}$$

$= (2)(-2) + (3)(3)$

$= -4 + 9 + 1$

$\boxed{\bar{T_1} \cdot \bar{T_2} = \cancel{\cos\theta} = 6}$

$|T_1| = \sqrt{(2)^2 + (3)^2 + (1)^2} = \sqrt{4+9+1}$

$= \sqrt{14}$

$|T_2| = \sqrt{(-2)^2 + (3)^2 + (1)^2}$

$= \sqrt{(4) + 9 + 1}$

$= \sqrt{14}$.

$$\cos\theta = \frac{\bar{T_1} \cdot \bar{T_2}}{|\bar{T_1}| \cdot |\bar{T_2}|}$$

$$= \frac{6}{\sqrt{14}\sqrt{14}}$$

$$= \frac{\cancel{6}\,3}{\cancel{14}\,7} = \boxed{\frac{3}{7}}$$

$$\boxed{\cos^{-1} = \frac{3}{7}}$$

2

Qu) Evaluate $\int_C \bar{F}\cdot d\bar{r}$ for $\bar{F}=(x^2+yz)\hat{i}+(y^2+xz)\hat{j}+(z^2+xy)\hat{k}$ along the straight line joining (0,0,0) to (1,1,1).

→ Given:-

$\bar{F}=(x^2+yz)\hat{i}+(y^2+xz)\hat{j}+(z^2+xy)\hat{k}$

$$\frac{x-0}{1-0}=\frac{y-0}{1-0}=\frac{z-0}{1-0}=t$$

$x=t,\ y=t,\ z=t$

$dx=dt,\ dy=dt,\ dz=dt.$

limit $t:\boxed{t=0}$ & $\boxed{t=1}$

line Integral $=\int_C \bar{F}\cdot d\bar{r}=\int_C F_1dx+F_2dy+F_3dz$

$$=\int_0^1[(t)^2+(t)(t)]dt+[(t)^2+(t)(t)]dt+[(t)^2+(t)(t)]dt$$

$$=\int_0^1[t^2+t^2]dt+[t^2+t^2]dt+[t^2+t^2]dt.$$

$$=\int_0^1 6t^2\,dt=\left[\frac{6t^3}{3}\right]_0^1=\frac{6(1)}{3}$$

$\therefore \boxed{\int_C \bar{F}\cdot d\bar{r}=2}$ //Ans

Q) Using Green's theorem evaluate $\int_C \bar{F}\cdot d\bar{r}$ for $\bar{F}=(4x+y)\hat{i}+(4y^2+5x)\hat{j}$ over the region R bounded by $y=0,\ x=1,\ y=x$.

→ Given:-

$\bar{F}=(4x+y)\hat{i}+(4y^2+5x)\hat{j}$.

$R:\ y=0,\ x=1,\ y=x.$

By Green's Theorem:

$$\oint_C \bar{F}\cdot d\bar{r}=\oint_C u\,dx+v\,dy=\iint_R\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dx\,dy \quad (1)$$

Here $\bar{F}=(4x+y)\hat{i}+(4y^2+5x)\hat{j}$

$=u\hat{i}+v\hat{j}$

where $u=4x+y$ $\quad v=(4y^2+5x)$

$\frac{\partial u}{\partial y}=1$ $\quad \frac{\partial v}{\partial x}=5$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=5-1=\boxed{4}$$

from (1)

$$\int_C \bar{F}\cdot d\bar{r}=\iint_R\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)dx\,dy$$

$$=\iint_R 4\,dx\,dy$$

$$=4\iint_R dx\,dy.$$

$$\int_C \bar{F}\cdot d\bar{r}=4\int_0^x\int_0^1 dx\,dy.$$

$= 4\int_0^x dy \cdot \int_0^1 dx$

$= 4[y]_0^x \cdot [x]_0^1$

$= 4[x-0] \cdot [1-0]$

$= 4[x] \cdot [1]$

$= \left[\frac{\cancel{4}^{2} x^2}{\cancel{2}_1}\right]_0^1$

$= [2x^2]_0^1$

$= 2(1)^2 - 2(0)$

$= \underline{\underline{2}}$ Ans

Q5) By stoke's theorem $\int_C \bar{F} \cdot d\bar{r}$ for $\bar{F} = (2x-y)\bar{i} - yz^2\bar{j} - zy^2\bar{k}$ over the surface $z = 4 - x^2 - y^2 \quad z > 0$

Ans Given:-

$\int_C \bar{F} \cdot d\bar{r}$ for $\bar{F} = (2x-y)\bar{i} - yz^2\bar{j} - zy^2\bar{k}$

$S: z = 4 - x^2 - y^2 \quad z > 0$

By Stoke's law:

$$\int_C \bar{F} \cdot d\bar{r} = \iint_S (\nabla \times \bar{F}) \cdot \hat{n} \, ds \quad -①$$

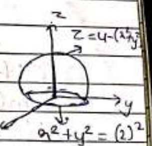
$$\nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (2x-y) & (-yz^2) & (-zy^2) \end{vmatrix}$$

$$= \bar{i}\left[\frac{\partial}{\partial y}(-zy^2) - \frac{\partial}{\partial z}(-yz^2)\right] - \bar{j}\left[\frac{\partial}{\partial x}(-zy^2) - \frac{\partial}{\partial z}(2x-y)\right] + \bar{k}\left[\frac{\partial}{\partial x}(-yz^2) - \frac{\partial}{\partial y}(2x-y)\right]$$

$$= \bar{i}[(-2zy) - (-2yz)] - \bar{j}[0-0] + \bar{k}[0-(-1)]$$

$$= \bar{i}[0] - \bar{j}[0] + \bar{k}(+1)$$

$$\boxed{\nabla \times \bar{F} = \bar{i} - \bar{j} + \bar{k}}$$



$S: z = 4 - x^2 - y^2 \quad z > 0$

$z = 4 - (x^2+y^2), \quad z > 0$

Here $\hat{n} = +\bar{k} = (0\bar{i} - 0\bar{j} + \bar{k})$

$(\nabla \times \bar{F}) \cdot \hat{n} = (-1)$

from ①

$$\int_C \bar{F} \cdot d\bar{r} = \iint_S (\nabla \times \bar{F}) \cdot \hat{n} \, ds$$

$$= \iint_S -1 \, ds$$

$$= -\iint_S ds$$

$\iint_S dx\,dy$

$R: x^2 + y^2 = 4.$

$=$ Area of circle $= x^2 + y^2 = (2)^2$

$$\boxed{\int \bar{F} d\bar{r} = \boxed{-4\pi}}$$

$-\frac{2\pi r^3}{3}$ $= 54\pi$

Q7) Using Guass Divergence Theorem, Eva...

$\iint_S \bar{F}\cdot\hat{n}$

$\rightarrow$ $\bar{F} = (y^2+x)\bar{i} + (z^2+y)\bar{j} + (x^2+z)\bar{k}$

$S: x^2+y^2+z^2 = 9.$

by Guass divergence theorem,

$$\iint_S \bar{F}\cdot\hat{n}\,ds = \iiint_V \nabla\cdot\bar{F}\cdot d\bar{v} \quad \text{---} ①$$

$$\nabla\cdot\bar{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= \frac{\partial}{\partial x}(y^2+x) + \frac{\partial}{\partial y}(z^2+y) + \frac{\partial}{\partial z}(x^2+z)$$

$$= 1+1+1$$

$\boxed{\nabla\cdot f = 3}$

from ①

$$\iint_S \bar{F}\cdot\hat{n}\cdot ds = \iiint_V \nabla\cdot\bar{F}\cdot d\bar{v}$$

$$\iiint_V (3)\,d\bar{v} = 3\iiint dx\,dy\,dz$$

$$= 3\,[\text{vol. of hemisphere}] = 3\left[\frac{2}{3}\pi r^3\right]$$

$$= \cancel{3}\left[\frac{2}{\cancel{3}}\times\pi\times 3\times 3\times 3\right] = 27\times 2 = \boxed{54\pi}$$

Vol. of sphere $= \frac{4}{3}\pi r^3$.

hemisphere $= \frac{2\pi r^3}{3}$

Q1) $\hat{u} = \frac{\bar{u}}{|\bar{u}|} = \frac{2\bar{i}+2\bar{j}+\bar{k}}{\sqrt{(2)^2+(2)^2+(1)^2}}$

$$\hat{u} = \frac{2\bar{i}+2\bar{j}+\bar{k}}{3}$$

$$D.D = [\nabla\phi]_P\cdot\hat{u}$$

$$= (\bar{i}+2\bar{j}+\bar{k})\cdot\frac{1}{3}(2\bar{i}+2\bar{j}+\bar{k})$$

$$= \frac{1}{3}(2+4+1) = \frac{1}{3}(7) = \boxed{\frac{7}{3} = D.D}$$

$\frac{f'(1)}{4}$

13/03/25

* Unit - V Numerical Methods *

* Solution or Algebraic and Transcedental Equation using Numerical Methods.

(1) Algebraic and Transcendental Equation:-
An Equation of the form,
$$F(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x + a_0 = 0$$
is called as algebraic Equation.

- Any non-algebraic Equation is called Transcendental Equation.

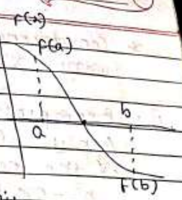
- Ex:- Algebraic. Eq^n
$x^3 + 2x^2 + 1 = 0$
$x^4 + x^2 + 1 = 0$
$3x + 1 = 0.$

- Ex:- Transcendal. Eq^n.
$x\cos x - 5 = 0$
$xe^x + 1 = 0.$
$\log x + 4\cos x = 0.$

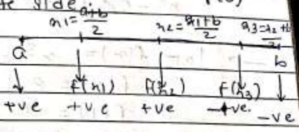
- Theorem:-
If for $F(x) = 0$ you can find two values of a & b of x such that $F(a)$ and $f(b)$ have opposite sides then,
$F(x) = 0$ has atleast one real root between a and b.

(1) Bisection Method:-

* procedure:-



1] Let $f(x) = 0$ be a given equation to be solved using bisection method first find two values a and b such that $F(a)$ & $f(b)$ have opposite side
$[f(a) < 0$ & $F(b) > 0]$



2] Here $F(x) = 0$ has at least one real root between a & b.

3] first approximation, say $x_1 = \frac{a+b}{2}$

4] find $f(x_1)$.

	If $f(x_1) < 0$	If $f(x_1) > 0$
1]	Here $F(x_1) < 0$ & $f(b) > 0$.	(1) Here $f(x_1) > 0$ & $f(a) < 0$
2]	Hence $F(x) = 0$ has at least one real root between x_1 & b.	2] Hence $F(x) = 0$ has at least One real root between x_1 & a.
3]	Second App $x_2 = \frac{x_1+b}{2}$	3] 2nd App $x_2 = \frac{x_1+a}{2}$
4]	find $f(x_2)$	4] find $f(x_2)$

5] Repeat above procedure to get best approximation of root.

* Convergence of bisection method is very slow:-

1) Example:-
Apply bisection method to find root of the equation,
$x^3 - 5x + 3 = 0$

→ let $f(x) = x^3 - 5x + 3$

a | b ve x f(x)

$f(0) = 3$ $\quad x = a, f(x) = f(a)$

$f(1) = -1$ $\quad x = b, f(x) = f(b)$

Here $a = 0$ and $b = 1$ such that $f(0) > 0$ and $f(1) < 0$

∴ Root lies between 0 & 1.

Table:

Approximation	a	b	f(a) (+ve)	f(b) (-ve)	$x = \frac{a+b}{2}$	f(x)
1st	0	1	3	-1	0.5	0.625
2nd	0.5	1	0.625	-1	0.875	-0.328
3rd	0.5	0.75	0.625	-0.328	0.625	0.119 ~~0.1~~
4th	0.625	0.75 ~~0.625~~	0.1191 ~~0.625~~	-0.328 ~~-0.123~~	0.6875 ~~0.5625~~	-0.1125 ~~0.23~~
5th	0.6625	0.6875	0.1191 ~~0.236~~	-0.1125 ~~-0.128~~	0.65625	+0.00[illegible]
6th	0.65625	0.6875	0.0013 1377	-0.1125	0.671875	-0.0560[illegible]
7th	0.65625	0.671875	0.00131377	-0.056079	0.6640625	-0.023[illegible]
8th	0.65629	0.6640625	0.00137329	-0.027 117	0.66017625	-0.0131[illegible]
9th	0.65629	0.6617625	0.00137329	-0.013084	0.658213	-0.0037 24

Approximation	a	b	f(a)	f(b)	$x = \frac{a+b}{2}$	f(x)
10th	0.65629	0.658213	0.00137329	~~-0.00226236~~ -0.013154	0.657231	-0.00226236
11th	0.65629	0.657231	0.00137329	~~-0.000685~~ -0.00226236	0.656806	-0.000685.
12th	0.65629	0.65680 510	0.00137 329.	-0.000 685	0.6565 4775	-0.000269406.
13th	0.6565 4775	0.65680 55	0.00269 400	-0.000 685	0.65667 6625	-0.0002082792
14th	0.6665 4775	0.6566 76625	0.000 2694000	-0.000 20829	0.6566 121875	0.0000080575.
15th	0.6566 121875	0.65667 662	0.0000 3055	-0.000 208275	0.6566 44406	-0.000208275.

• Hence a root of the equation $x^3 - 5x + 3 = 0$ correct to four decimal places is 0.6566.

Q1) Apply bisection method to find the root of the equation:

$x^3 - x^2 - x + 1 = 0$ correct upto 3 decimal places

Ans:-
Let,
$f(x) = x^3 - x^2 - x + 1 = 0$ $\quad$ ~~f(-1) = 2~~.
$f(0) = -1$
$f(1) = -1$
$f(2) = 5$
~~f(4) = 54~~.

48 − 4 = 44, 44 + 1 = 45

Here $a = 1$ & $b = 2$
∴ $f(1)$ & $f(2)$ have opposite signs.

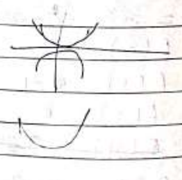
$\therefore f(x) = 0$ has at least one real root between 1 & 2.

-ve +ve

Approximation	a	$x = \frac{a+b}{2}$	b	f(a)	f(x)	f(b)
1st	1	1.5	2	-1	0.875	5
2nd	1	1.25	1.5	-1	-0.296875	0.875
3rd	1.25	1.375	1.5	-0.296875	0.2246	0.875
4th	1.25	1.3125	1.375	-0.296875	-0.05151	0.2246
5th	1.3125	1.34375	1.375	-0.0515	0.0126	0.2246
6th	1.3125	1.328125	1.34375	-0.0575	0.01457	0.0826
7th	1.3125	1.3203125	1.328125	-0.0515	-0.0187	0.01457
8th	1.3203125	1.32421875	1.328125	-0.0187	-0.00244	0.01457
9th	1.32421875	1.326171875	1.328125	-0.002	0.0062	0.01457

$y = a x^2$

$y = -a x^2$



- Polar coordinat
- Area of cartesian coordinate
- double integration, with limit & without limit.

Imp

Unit 3

① Evaluate $\iint_R (y+2x^2)\,dx\,dy$ over the region bounded by $y = x^2$ & $2y - x = 0$.

② Evaluate $\iint_R (xy + x^2)\,dx\,dy$ over the region bounded by triangle, $y = x$, $x = 2$, $y = 0$.

③ Evaluate $\iint_R (x - y^2)\,dx\,dy$ over the region bounded by $x = y^2$, $y = 0$, $x + y - 1 = 0$.

Unit-4

① find the area bounded by the curve $r = 2\cos\theta$ above the initial line.

② find the area of cardioid, $r = a(1-\cos\theta)$.

③ find the area bounded by outside the circle $r = 2a\sin\theta$ and inside the cardioid $r = a(1+\sin\theta)$

① Evaluate :- $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} xyz\,dx\,dy\,dz$.

② Evaluate :- $\int_{0}^{1}\int_{1}^{y^2}\int_{0}^{1-x} (x+y+z)\,dx\,dy\,dz$.

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(3) Evaluate:-
$$\int_0^2\int_0^x\int_0^{x+y} (x^2+yz)\,dx\,dy\,dx.$$

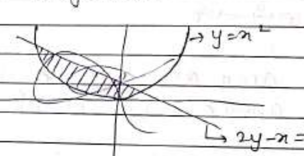
Answer:-

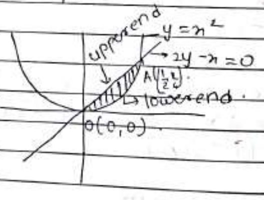
(1)

→ let $I = \iint_R (y+2x^2)\,dx\,dy$

$R: y=x^2$ & $2y-x=0$ $x=2y$.

· Region Integration:-





· point of Intersection:-
$y=x^2$ and $y=\cancel{x}\,2y-x$.
$2x^2-x=0$
$\cancel{2x(x-1)}=0$
$\cancel{2x}\ x(2x-1)=0$
$x=0$ $\quad 2x-1=0$
$\boxed{x=\frac{1}{2}}$

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limit:
$y=x^2$, $y=x/2$
$x=0$, $x=1/2$.

$$\int_{x=0}^{1/2}\int_{y=0}^{1/4}(y+2x^2)\,dx\,dy.$$

$$\rightarrow \int_{x=0}^{1/2}\left[\int_{y=0}^{1/4}(y+2x^2)\,dy\right]dx.$$

$$= \int_{x=0}^{1/2}\left[\left[\frac{y^2}{2}+2x^2y\right]_{y=0}^{1/4}\right]dx$$

$$= \int_{x=0}^{1/2}\left[\frac{\left(\frac{1}{4}\right)^2}{2}+2x^2(1/4)\right]dx$$

$$= \int_{x=0}^{1/2}\left[\frac{\frac{1}{16}}{2} + 2x^2\left(\frac{1}{4}\right)\right]dx$$

$$= \int_{x=0}^{1/2}\left[\frac{1}{8} + 2x^2\frac{1}{4}\right]dx$$

$$= \left[\frac{1}{8}x+\frac{x^3}{3}\frac{1}{4}\right]_0^{1/2}$$

$$= \left[\frac{1}{8}\left(\frac{1}{2}\right)+\frac{\left(\frac{1}{2}\right)^3}{3}\frac{1}{4}\right]$$

$$= \frac{1}{16}+\frac{\frac{1}{8}}{3}\cdot\frac{1}{4}$$

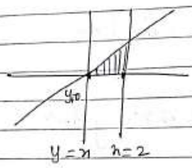
$$= \frac{1}{16}+\frac{\frac{1}{2}}{3} = \frac{1}{16}+\frac{3}{2} = \frac{1}{16}+\frac{3\times 8}{2\times 8} = \frac{1}{16}+\frac{24}{16} = \boxed{\frac{25}{16}}$$

$\frac{2}{960}$

Q2) let $I = \iint_R (xy + x^2)\, dx\, dy$.

$R: y = x,\ x = 2,\ y = 0.$

- Region of Integration:-



- point of Intersection:-
  limit of intersection,
  limit: $y = x$, $y = 0$
  limit: $x = 2$, $x = 0$

$$= \int_{x=0}^{2}\int_{y=0}^{x} (xy + x^2)\, dx\, dy.$$

$$= \int_{x=0}^{2}\left[\int_{y=0}^{x}\left[x \cdot \frac{y^2}{2} + x^2 y\right] dy\right] dx.$$

$$= \int_{x=0}^{2}\left[\left[x \cdot \frac{y^2}{2} + x^2 y\right]_0^x dy\right] dx$$

$$= \int_{x=0}^{2}\left[x \cdot \frac{(x)^2}{2} + x^2(x) - \cancel{x \cdot \frac{(0)^2}{2} + x^2}\right] dx$$

$$= \int_{x=0}^{2}\left[x \cdot \frac{x^2}{2} + x^3\right] dx$$

$2\times2\times2\times2$

$$= \left[\frac{x^2}{2} \cdot \frac{x^3}{3} + \frac{x^4}{4}\right]_0^2$$

$$= \left[\frac{(2)^2}{2} \cdot \frac{(2)^3}{3} + \frac{(2)^4}{4}\right]$$

$$= \frac{4^2}{2} \cdot \frac{8}{3} + \frac{16}{4}$$

$$= \cancel{2 \cdot \frac{8}{3} + \frac{8}{3}}\ 2 \cdot \frac{8}{3} + 4.$$

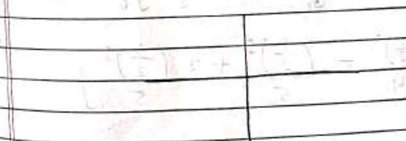
$$= \boxed{6 \text{ units}}$$

$\pi/6$

Q3) let $I = \iint_R (x - y^2)\, dx\, dy$.

$R: x = y^2,\ y = 0,\ x + y - 2 = 0.$

- Region of Intersection:-



(Q1) limit

$y = x^2, \quad y = x/2$

$x = 0, \quad x = 1/2.$

$$\rightarrow \int_{x=0}^{1/2}\int_{y=x^2}^{x/2} (y + 2x^2)\, dx\, dy.$$

$$\rightarrow \int_{x=0}^{1/2}\left[\int_{x^2}^{x/2} y + 2x^2\, dy\right] dx$$

$$= \int_{0}^{1/2}\left[\frac{y^2}{2} + 2x^2 y\right]_{x^2}^{x/2} dx$$

$$= \int_{0}^{1/2}\left[\frac{\left(\frac{x}{2}\right)^2}{2} + 2x^2\left(\frac{x}{2}\right) - \frac{(x^2)^2}{2} + 2x^2(x^2)\right] dx$$

$$= \int_{0}^{1/2}\left[\frac{\frac{x^2}{4}}{2} + \frac{2x^3}{2} - \frac{x^4}{2} + 2x^4\right] dx.$$

$$= \int_{0}^{1/2}\left[\frac{x^2}{2} + \frac{x^3}{1} - \frac{x^4}{2} + 2x^4\right] dx.$$

$\frac{16}{2}$, 32

$$= \left[\frac{x^3}{3} + \frac{x^4}{4} - \frac{x^5}{5} + 2\frac{x^5}{5}\right]_0^{1/2}$$

$\frac{32}{2}$, 64

$$= \left[\frac{\left(\frac{1}{2}\right)^3}{3} + \frac{\left(\frac{1}{2}\right)^4}{4} - \frac{\left(\frac{1}{2}\right)^5}{5} + 2\frac{\left(\frac{1}{2}\right)^5}{5}\right]$$

$$= \left[\frac{\frac{1}{8}}{3} + \frac{\frac{1}{16}}{4} - \frac{\frac{1}{32}}{5} + 2\frac{\frac{1}{32}}{5}\right]$$

$$\frac{\frac{1}{8}}{3} + \frac{1}{4} - \frac{5}{32} + \frac{64}{5}\Big]$$

$$= \left[\frac{3}{8} + \frac{1}{4} - \frac{5}{32} + \frac{64}{5}\right]$$

$$= \frac{2123}{160}$$

21/03/25

Unit-4

(Q1) find the angle between tangent to the curve,
$x = t^2, y = t+1, z = t^3 - 1$ at $t=1$ & $t=2$.

Q2) find the angle between tangent to the curve.

$x = t+1, y = 2t^3, z = t^2 - 1$ at $t=0$ & $t=1$.

Q3) Show that the vector field.
$\bar{F} =$

Answer,

Q1)
→ Given curve:-
i's $x = t^2, y = t+1, z = t^3 - 1$
position vector $= \bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$\bar{r} = (t^2)\bar{i} + (t+1)\bar{j} + (t^3-1)\bar{k}$

Tangent vector $= T = \dfrac{d\bar{r}}{dt}$

$\bar{T} = (2t)\bar{i} + (t)\bar{j} + (3t^2)\bar{k}$

At $t=1$

$\bar{T} = 2(1)\bar{i} + (1)\bar{j} + (3(1)^2)\bar{k}$

$\boxed{\bar{T} = 2\bar{i} + \bar{j} + 3\bar{k}}$

At $t=2$

$\bar{T} = 2(2)\bar{i} + (2)\bar{j} + (3(2)^2)\bar{k}$

$\boxed{T_2 = 4\bar{i} + 2\bar{j} + 12\bar{k}}$

Angle between:-

$\cos\theta = \dfrac{T_1 \cdot T_2}{|T_1| \cdot |T_2|}$ — ①

$= \dfrac{(2\bar{i} + \bar{j} + 3\bar{k}) \cdot (4\bar{i} + 2\bar{j} + 12\bar{k})}{\sqrt{(2)^2 + (1)^2 + (3)^2} \cdot \sqrt{(4)^2 + (2)^2 + (12)^2}}$

$= \dfrac{(2)(4) + (1)(2) + (3)(12)}{\sqrt{4+1+9} \cdot \sqrt{16+4+144}}$

$= \dfrac{8 + 2 + 36}{\sqrt{14}\ \sqrt{164}}$

$= \boxed{\dfrac{46}{\sqrt{14}\sqrt{164}}}$ //

12
12
24
120
144
4
148
16
164

36
8
44
2
46

(Q2) Given:-
$x = t+1,\ y = 2t^3,\ z = t^2 - 1$ at $t = 0, t = 1$

→ position vector:
$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

Tangent vector $\bar{T} = \frac{d\bar{r}}{dt}$

$\bar{T} = (1)\bar{i} + (6t^2)\bar{j} + (2t)\bar{k}$.

at,
$t = 0,\ t = 1$

At $t = 0$
$\bar{T} = (0)\bar{i} + (6(0)^2)\bar{j} + (2(0))\bar{k}$
$\bar{T} = \cancel{0\bar{i}} + 0 + 0\bar{i}$

At $T = 1$
$\bar{T} = (1)\bar{i} + (6(1)^2)\bar{j} + (2(1))\bar{k}$
$\boxed{\bar{T} = 1\bar{i} + 6\bar{j} + 2\bar{k}}$

Angle,
$\cos\theta = \frac{\bar{T_1} \cdot \bar{T_2}}{|T_1|\,|T_2|}$

$= \frac{(0\bar{i}) \cdot (1)\bar{i} + 6\bar{j} + 2\bar{k}}{\sqrt{0} \cdot \sqrt{(1)^2 + (6)^2 + (2)^2}}$

$= 0\bar{i} \cdot \frac{1\bar{i} + 6\bar{j} + 2\bar{k}}{\sqrt{1 + 36 + 4}}$

$= \boxed{\frac{\bar{0}\,1}{\sqrt{41}}}$

(Q3)
Let $I = \bar{F} = (6xy + z^3)\bar{i} + (3x^2 - z)\bar{j} + (3xz^2 - y)\bar{k}$
Comparing with,
$\bar{F} = f_1\bar{i} + f_2\bar{j} + f_3\bar{k}$
where $f_1(6xy + z^3)\bar{i} + f_2(3x^2 - z)\bar{j} + f_3(3xz^2 - y)\bar{k}$

To find curl $\bar{F}$

$$\nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (6xy + z^3) & (3x^2 - z) & (3xz^2 - y) \end{vmatrix}$$

$$= \bar{i}\left(\frac{\partial}{\partial y}(3xz^2 - y) - \frac{\partial}{\partial z}(3x^2 - z)\right) - \bar{j}\left(\frac{\partial}{\partial x}(3xz^2 - y) - \frac{\partial}{\partial z}(6xy + z^3)\right) + \bar{k}\left(\frac{\partial}{\partial x}(3x^2 - z) - \frac{\partial}{\partial y}(6xy + z^3)\right)$$

$= \bar{i}[(-1) - (-1)] - \bar{j}[3z^2] - [3z^2] + \bar{k}[6x - 6x]$

$= \bar{i}[0] - \bar{j}[0] + \bar{k}[0]$

$\boxed{\nabla \times F = 0}$

$\phi = \int f_1\,dx + \int_{x = \text{free}} f_2\,dy + \int_{x,y\ \text{free}} f_3\,dz + C.$

$$\phi = \int (6xy + z^3)\,dx + \int_{x=\text{free}} (3x^2 - z)\,dy + \int_{x,y=\text{free}} (3xz^2 - y)\,dz.$$

$$\phi = \int (6x^2y + xz^3) + \int (-zy) + \int$$

$$\boxed{\phi = 3x^2y + xz^3 - zy + C}$$

Qu]

$\bar{F} = (yz - x^2)\bar{i} + (2x - y^2)\bar{j} + (xy - z^2)\bar{k}$

Comparing with,

$\bar{F} = f1\,\bar{i} + f2\,\bar{j} + f3\,\bar{k}$

where; $f1 (yz - x^2)\,\bar{i}$, $f2 (2x - y^2)\,\bar{j}$, $f3\,(xy - z^2)\bar{k}$

To find curl $\bar{F}$.

$$\nabla \times \bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f1 & f2 & f3 \end{vmatrix}$$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (yz - x^2) & (2x - y^2) & (xy - z^2) \end{vmatrix}$$

$$= \bar{i}\left(\frac{\partial}{\partial y}(xy - z^2) - \frac{\partial}{\partial z}(2x - y^2)\right) - \bar{j}\left(\frac{\partial}{\partial x}(xy - z^2) - \frac{\partial}{\partial z}(yz - x^2)\right) + \bar{k}\left[\frac{\partial}{\partial x}(2x - y^2) - \frac{\partial}{\partial y}(yz - x^2)\right]$$

$$= \bar{i}\left[(x) - (0)\right] - \bar{j}\left[(y) - (y)\right] + \bar{k}\left[(2) - (z)\right]$$

$$\boxed{\nabla \times \bar{F} = 0}$$

$$\phi = \int f1\,dx + \int_{x\text{ free}} f2\,dy + \int_{x,y\text{ free}} f3\,dz.$$

$$= \int (yz - x^2)\,dx + \int_{x\text{ free}} (2x - y^2)\,dy + \int_{x,y\text{ free}} (xy - z^2)\,dz.$$

$$\boxed{\phi = xyz - \frac{x^3}{3} - y^2 - z^2}$$

Q5] Using Guass Divergence Theorem, evaluate $\iint_S \bar{F}\cdot\hat{n}\,ds$ over the Surface bounded by $0 \le x \le 1$, $0 \le y \le 2$, $0 \le z \le 3$ &

$\bar{F} = (y^2 + zx z)\,\bar{i} + (z^2 + 2xy)\,\bar{j} + (x^2 + 2yz)\,\bar{k}$

Q6] Using Guass Divergence theorem, Evaluate $\iint_S [x^2\bar{i} + y^2\bar{j} + z^2\bar{k}]\,d\bar{s}$ over the Surface bounded by,

$x = 0$, $x = 2$

$y = 0$, $y = 2$

$z = 0$, $z = 2$.

Q1) $\bar{F} = (y^2+2xz)\bar{i} + (z^2+2xy)\bar{j} + (x^2+2yz)\bar{k}$

$S: x=0, x=1, y=0, y=2, z=0, z=3.$

By Guass theorem,

$$\iint_S \bar{F}\cdot\hat{n}\,ds = \iiint_S \nabla\cdot\bar{F}\,dv \quad -(1)$$

Now, $\nabla\cdot\bar{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

$= \frac{\partial}{\partial x}(y^2+2xz) + \frac{\partial}{\partial y}(z^2+2xy) + \frac{\partial}{\partial z}(x^2+2yz)$

$\nabla\cdot\bar{F} = 2z + 2x + 2y.$

from eq^n (1)

$$\iint_S \bar{F}\cdot\hat{n}\,ds = \iiint_V \nabla\cdot\bar{F}\,dv$$

$$= \iiint_V (2z+2x+2y)\,dv$$

$$= \int_{z=0}^{3}\int_{y=0}^{2}\int_{x=0}^{1} (2z+2x+2y)\,dv.$$

$$= \int_0^2\int_0^1 \left[2\frac{z^2}{2} + 2xz + 2yz\right]_0^3 dz$$

$$= \int_0^2\int_0^1 \left[2\frac{(3)^2}{2} + 2(x)(3) + 2y(3)\right]$$

$$= \int_0^2\int_0^1 \left[\cancel{2}\frac{(9)}{\cancel{2}} + 6x + 6y\right]$$

$$\int_{y=0}^{2}\int_{x=0}^{1} \left[9 + 6x + 6y\right] dy\,dx$$

$$\int_0^1 \left[9y + 6xy + 6\frac{y^2}{2}\right]_0^2 dx$$

$$\int_0^1 \left[9(2) + 6x(2) + 6\frac{(2)^2}{2}\right] dx$$

$$\int_0^1 \left[18 + 12x + \frac{\cancel{6}3(4)}{\cancel{2}1}\right] dx$$

$$\int_0^1 \left[18 + 12x + 12\right] dx$$

$$\left[18x + 12\frac{x^2}{2} + 12x\right]_0^1$$

$18(1) + \frac{\cancel{12}^{6}(1)^2}{\cancel{2}_1} + 12(1)$

$18 + 6 + 12$

$= \underline{\underline{36}}$

$\begin{array}{r}18\\12\\6\\\hline 36\end{array}$

25/03/25

* Find the root of the equation, $x^3-4x+1=0$ line in the interval, $\left[0, \frac{1}{2}\right]$ by bisection method correct upto 3 decimal places.

Solution: Given:

$f(x) = x^3-4x+1$.

$a=0$ $\quad f(0)=1$

$b=\frac{1}{2}$ $\quad f\left(\frac{1}{2}\right) = -\frac{7}{8}$

$\therefore$ f(0) and $f(1/2)$ have opposite signs.

$\therefore f(x)=0$ has at least one real root between 0 and $\frac{1}{2}$.

Approximation	a	x=a+b/2	b	f(a)	f(x)	f(b)
1st	0	0.25	0.5	1	0.015625	-0.875
2nd	0.25	0.375	0.5	0.015625	-0.427	-0.875
3rd	0.25	0.3125	0.375	0.015625	-0.21948	0.40126
4th	0.25	0.28125	0.3125	0.015625	-0.10275	-0.21948
5th	0.25	0.265625	0.28125	0.015625	-0.04375	-0.10275
6th	0.25	0.2578125	0.265625	0.015625	-0.14113	0.04375
7th	0.25	0.25390625	0.2578125	0.015625	0.00074487	-0.014113
8th	0.25390625	0.2558937	0.2578125	0.00074487	-0.006687	-0.014113
9th	0.25390625	0.2548824	0.2558937	0.00074487	-0.0029727	-0.006687
10th	0.2539065	0.254394	0.2548828125	0.00074487	-0.0011145	0.00297272
11th	0.25390625	0.254153	0.254394	0.00074487		0.0011145

• Hence 0.254 is the real root correct to three decimal places.

2) Find the root of the equation, $x^3-4x-9=0$ line in the interval, [2, 3] using bisection method correct up to 3 decimal places.

Ans: Given:

$f(x) = x^3-4x-9$

$a=2$ $\quad f(2)=-9$.

$b=3$ $\quad f(3)=6$.

f(2) and f(3) have opposite signs.

$\therefore f(x)=0$ has at least one real root between 2 and 3.

Approximation	a (-ve)	$x=\frac{a+b}{2}$	b (+ve)	f(a) (-ve)	f(x)	f(b) (+ve)
1st	2	2.5	3	-9	-3.375	6.
2nd	2.5	2.75	3	-3.375	0.796875	6
3rd	2.5	2.625	2.75	-3.375	-1.412109	0.796875
4th	2.625	2.6875	2.75	-1.412109	-0.339111	0.796875
5th	2.6875	2.71875	2.75	-0.339111	0.220916	0.796875
6th	2.71875	2.703125	2.71875	-0.339111	-0.061077	0.220916
7th	2.703125	2.7109375	2.71875	-0.061077	0.07942	0.220916
8th	2.703125	2.70703125	2.7109375	-0.061077	0.009049	0.07942
9th	2.703125	2.705078125	2.70703125	-0.061077	-0.026044	0.009049
10th	2.705078125	2.70605	2.70703125	-0.0260448	-8.5897	0.009049

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(Q) find the root of the equation, $x^3 - 2x - 5 = 0$ using bisection up to 3 decimal

→ Given:

$f(x) = x^3 - 2x - 5 = 0$

$a = 2$ $f(a) = -1$

$b = 3$ $f(b) = 16$.

$f(2)$ and $f(3)$ have opposite signs.

∴ $f(x) = 0$ has at least one real root between 2 and 3.

Approximation	a	$x = \frac{a+b}{2}$	b	f(a)	f(x)	f(b)
1st	2	2.5	3	-1	5.625	16
2nd	~~2.5~~ 3.	2.75	2.5	~~5.625~~ -1	10.296875	~~16~~ 5.625
3rd	3	2.875	2.75	-1	13.0136718	10.296875
4th	3	2.9375	2.875	-1	14.47241211	13.01361
5th	3	2.96875	2.9375	-1	15.22750814	14.472412

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Q) find the root of the equation, ~~$2x - \cos x - 3 = 0$~~ $x - e^{-x} = 0$, $x \in [0.5, 1]$ using bisection method correct up to 3 decimal places.

Ans. Given:

$f(x) = x - e^{-x}$, $x \in [0.5, 1]$

$a = 0.5$ $f(0.5) = -0.1065$

$b = 1$ $f(1) = 0.6321$.

∴ $f(0.5) < 0$ & $f(1) > 0$

∴ The root $f(x) = 0$ has at least one real root between 0.5 & 1.

Approximation	a	$x = \frac{a+b}{2}$	b	f(a) (-ve)	f(x)	f(b) (+ve)
1st.	0.5	0.75	1	-0.1065	0.277633 4473	0.6321
2nd	0.5	0.625	0.75	-0.1065	0.08973 57145	0.277633 4473
~~2nd~~ 3rd	0.5	0.5625	0.625	-0.1065	-1.19255 4657	0.08973 857148
4th	0.5625	0.59375	0.625	~~-1.19255 4657~~	-1.2170160 72	0.08973 857148
5th	0.59375	0.609375	0.625	-0.00732 -1.2170160	-1.229906 459	0.08973 857148
6th	0.609375	0.6171875	0.625	-1.22990 6489	-1.2365196 57	0.08973 857148
7th	0.6171875	0.62109375	0.625	-1.23651 9657	-1.23986 86 07	0.08973 857148
8th				-1.2398686 07		0.08973 857148

0.451

Q) Given:

$f(x) = x^2 + x - \cos x = 0 \quad x \in (0, 1)$

$a = 0 \quad f(0) = -1$

$b = 1 \quad f(1) = 1.00018$

Approximation	a (-ve)	$x = \frac{a+b}{2}$	b (+ve)	f(a) (-ve)	f(x)	f(b) (+ve)
1st	0	0.5	1	-1	-0.249961 9231	1.00018
2nd	0.5	0.75	1	-0.249961 9231	0.3125856 724.	1.00018
3rd.	0.5	0.625	0.75	-0.249961 9231	0.01568 4495.	0.3125856 724
4th	0.5	0.5625	0.625	-0.249961 9231	-0.12104 5559.	0.01568 4495
5th	0.5625 ~~0.625~~	0.59375	0.625	-0.12104 5559	-0.05365 724321	0.01568 4495
6th	0.59375	0.609375	0.625	-0.05365 724321	-0.019230 55191	0.01568 4495
7th.	0.609375	0.6171875	0.625	-0.019230 55191	-0.00183 ~~-1.834072 909~~.	0.01568 4495
8th	0.6171875	0.62109375	0.625	-0.00183	-0.00691	0.01568 4495
9th	0.62109375	0.623046 875	0.625	-0.00691	0.011293 40719	0.01568 4495
10th						

Q. Newton - Raphson Method.

formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$n = 0, 1, 2, 3 \ldots$

x_0 = initial approximation.

- Use Newton - Raphson Method to find root of the equation.

$x^3 - 4x - 9 = 0, \quad x \in [2, 3]$.

→ Let $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$.

Here $f(2) = -9$ and $f(3) = 6$.

∴ $f(x) = 0$ has at least one real root between 2 & 3.

By Newton - Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad n = 0, 1, 2, 3 \ldots$$

1st Intial Approximation,

Take $x_0 = 2.5$

first Approximation, (put $n=0$):

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2.5 - \frac{f(2.5)}{f'(2.5)}$$

$$= 2.5 - \frac{(-3.375)}{14.75}$$

$\boxed{x_1 = 2.72881}$

- 2nd Approximation (put n=1)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.72881 - \frac{f(2.72881)}{f'(2.72881)}$$

$$= 2.72881 - \frac{0.404581}{18.339}$$

$\boxed{x_2 = 2.7068}$

- 3rd Approximation (put n=2)

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7067 - \frac{f(2.7067)}{f'(2.7067)}$$

$$= 2.7067 - \frac{0.003092}{17.9786}$$

$\boxed{x_3 = 2.7065}$

- 4th Approximation (put n=3)

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.706 - \frac{f(2.7065)}{f'(2.7065)}$$

$$= 2.706 - \frac{00.0050250}{17.96}$$

$\boxed{x_4 = 2.7059}$

* Use Newton - Raphson method to find root of the equation: $x^3 - 5x + 3 = 0$.

→ Let $f(x) = x^3 - 5x + 3$

$f'(x) = 3x^2 - 5$.

Here $f(0) = 3$, $f(1) = -1$

∴ f_0 and $f(1)$ have opposite signs. Hence $f(x) = 0$ has at least one real root between 0 and 1.

- By Newton - Raphson Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad n = 0, 1, 2, \ldots$$

- Initial Approximation,

Take $x_0 = 0.7$

first Approximation (put n=0)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.7 - \frac{f(0.7)}{f'(0.7)}$$

$$= \cancel{0.7 - 0.625} \quad 0.7 - \frac{(-0.157)}{-3.53}$$

$\boxed{x_1 = 0.6555240793}$

2nd Approximation: (put $n=1$)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.6555 - \frac{0.004155}{-3.7109}$$

$$\boxed{x_2 = 0.6566}$$

- 3rd approximation put $n=2$.

$$x_3 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = 0.6566 - \frac{0.00075 72}{-3.7066}$$

$$\boxed{x_3 = 0.6566}$$

∴ Hence root of the equation $x^3-5x+3=0$ correct to then decimal places is 0.6566.

(Q) Find the approximate value of $\sqrt{28}$ correct upto 3 decimal places using newton raphson method.

→ let $x = \sqrt{28} \Rightarrow \sqrt{25}$ & $\sqrt{36}$

5 ← 28 → 6

$x^2 = 28 \Rightarrow x^2 - 28 = 0$

$f(x) = x^2 - 28$

$f'(x) = 2x_0$

$f(5) = -3$ $\quad f(6) = -8$.

$f(5)$ and $f(6)$ have opposite signs hence $f(x)=0$ has at least one real root between 0 and 1.

- By Newton - Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad n = 0, 1, 2, \ldots$$

- first approximation (put $n=0$)

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5.5 - \frac{f(5.5)}{f'(5.5)}$$

$$5.5 - \frac{2.25}{11}$$

$$\boxed{x_1 = 5.295}$$

2nd approximation (put $n=1$)

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 5.295 - \frac{f(5.295)}{f'(5.295)}$$

$$= 5.295 - \frac{0.037025}{10.59}$$

$$\boxed{x_2 = 5.291}$$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 5.2915 - \frac{f(5.2915)}{f'(5.2915)}$

$= 5.2915 - \frac{0.0370}{10.582}$

$\boxed{x_3 = 5.288} = 5.291$

(9) Find the approximate value of $\sqrt[3]{13}$ correct upto 3 decimal places using Newton - Raphson method.

→ Let $x = \sqrt[3]{13}$

$x^3 = 13 \Rightarrow x^3 - 13 = 0.$

$f(x) = x^3 - 13$

$f'(x) = 3x^2.$

Here $f(2) = -5$ $\quad f(3) = 14.$

- By N-R

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x(0) - \frac{f(0)}{f'(0)}.$

$= \frac{-13}{0}$

$x_1 = 2.5 - \frac{2.625}{18.75}$

$\boxed{x_1 = 2.36}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.36 - \frac{f(2.36)}{f'(2.36)}.$

$= 2.36 - \frac{0.1442}{16.7088}$

$\boxed{x_2 = 2.35136}$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.35136 - \frac{f(2.35136)}{f'(2.35136)}$

$= 2.35136 - \frac{0.0004198}{16.5866}$

$\boxed{x_3 = 2.3513}$ //

① Find the value of $\sqrt[3]{\frac{1}{4}}$ correct up to 3 decimal places using N-R method.

② Find the root of the equation $e^{-x} - x = 0$ between 0 and 1 correct up to three decimal places.

③ ~~Find~~ By N-R method find a root of the equation $e^{-x} - \sin x = 0$ between 0 & 1 correct upto 3 decimal places.

(4) By N-R method find a root of the equation $\cos x - x = 0$ between 0.5.

Answer

(1)

Ans:→ let $x = \sqrt[3]{\frac{1}{4}}$

$x^3 = \frac{1}{4} = x^3 - \frac{1}{4} = 0$

$f(x) = x^3 - \frac{1}{4}$

$f'(x) = 3x^2$

Here $f(0) = -\frac{1}{4}$ $f(1) = \frac{3}{4}$.

- By N-R method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x(0.5) - \frac{f(0.5)}{f'(0.5)}$

$x_1 = 0.5 - \frac{(-0.125)}{0.75}$

$\boxed{x_1 = 0.666}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.666 - \frac{f(0.666)}{f'(0.666)}$.

$= 0.666 - \frac{0.04540}{1.330668}$

$\boxed{x_2 = 0.631}$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.631 - \frac{f(0.631)}{f'(0.631)}$

$= 0.631 - \frac{0.00123}{1.194483}$

$\boxed{x_3 = 0.6299}$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 0.6299 - \frac{f(0.629)}{f'(0.629)}$

$= 0.6299 - \frac{(-0.001141)}{1.186923}$

$\boxed{x_4 = 0.63086}$ → Ans.

Q (2) Ans let $f(x) = e^{-x} - x = 0$

$f'(x) = -e^{-x} - 1 = 0$

$f(0) = -1 - 1 = -2$.

$f(1) = -0.632$

$f(-1) = 3.718281$.

- By N-R method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x(-2) - \frac{f(-2)}{f'(-2)}$$

$$x_1 = x(-2) - \frac{9.38905}{-5.389056}$$

$$\boxed{x_1 = -0.257756}$$ //

$$x_1 = -0.5 - \frac{f(-0.5)}{f'(-0.5)}$$

$$= -0.5 + \frac{(2.1487)}{\cancel{1.148727}\ 2.648721}$$

~~$x_1 = 1.37050$~~

0.567

$$= -0.5 + \frac{(2.1487)}{2.648721}$$

$$\boxed{x_1 = 0.31122}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.31122 - \frac{(0.31122)}{f'(0.31122)}$$

$=$

28/09/15

2) Interpolation:-

* Suppose we are given following values of $y=f(x)$ for a set of values of x

$x =$	x_0	x_1	x_2	....	x_n
$y=f(x)$	y_0	y_1	y_2	....	y_n

- The process of finding the value of y corresponding to any value of $x = x_i$ between x_0 and x_n is called Interpolation.

- Thus Interpolation is a technique of estimating the values of a function for any intermediate value of the independent variable.

1) Langranges' Interpolation formula:

- There are many Interpolation formulas are available only for equally space values of the argument x.

- Langranges Interpolation formula is for unequally space argument value.

- If $y=f(x)$ take the values $y_0, y_1, y_2, \dots y_n$ corresponding to $x_0, x_1, x_2, \dots x_n$, then polynomial passing through given point is given by,

$$y = p_n(x) = \sum_{i=0}^{n} y_i l_i(x)$$

where, $l_i(x) = \dfrac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)}$

$$y = y_0 l_0(x) + y_1 l_1(x) + y_2 l_2(x) + \dots + y_n l_n(x).$$

(l_0 = leave)

$$y = y_0\left[\frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\dots(x_0-x_n)}\right] +$$

(x_1 = leave)

$$y_1\left[\frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\dots(x_1-x_n)}\right] + \dots$$

[x_n = leave]

$$+ y_n\left[\frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})}\right]$$

- This is know langranges Interpolation formula for unequal Interval.

Note: langranges Interpolation formula can also be use for equally space value of x.

- Ex:-

Q) Find langranges Interpolating Polynomial passing through set of point.

x	0	1	2
y	4	3	6

Use it to find,
1) y at $x = 1.5$
2) $\dfrac{dy}{dx}$ at $x = 0.5$
3) $\int_0^3 y\,dx$.

→ Given:

	x_0	x_1	x_2
x	0	1	2
y	4	3	6
	y_0	y_1	y_2

By lagrange's Intepretation formula,

$$y = P_n(x) = \sum_{i=0}^{n} y_i L_i(x)$$

$$y = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x).$$

$$y = y_0\left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right] + y_1\left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right] + y_2\left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right]$$

Given:

$x_0 = 0$, $x_1 = 1$, $x_2 = 2$

$y_0 = 4$, $y_1 = 3$, $y_2 = 6$

$$y = 4\left[\frac{(x-1)(x-2)}{(0-1)(0-2)}\right] + 3\left[\frac{(x-0)(x-2)}{(1-0)(1-2)}\right] + 6\left[\frac{(x-0)(x-1)}{(2-0)(2-1)}\right]$$

$$= \cancel{4}^{2}\left[\frac{x^2-3x+2}{\cancel{2}}\right] + 3\left[\frac{x^2-2x}{(-1)}\right] + \cancel{6}^{3}\left[\frac{x^2-x}{\cancel{2}}\right]$$

$$= 2(x^2-3x+2) + (-3)[x^2-2x] + 3(x^2-x)$$

$$= 2x^2 - \cancel{6x} + 4 - \cancel{3x^2} + \cancel{6x} + \cancel{3x^2} - 3x$$

$$\boxed{y = 2x^2 - 3x + 4.}$$

① y at $x = 1.5$

$[y]_{x=1.5} = 4.$

② $y = 2x^2 - 3x + 4,$

$\frac{dy}{dx} = 4x - 3$

$$\boxed{\left[\frac{dy}{dx}\right]_{x=0.5} = -1}$$

③ $$\int_0^3 y\,dx = \int_0^3 (2x^2 - 3x + 4)\,dx$$

$$= \left[\frac{2x^3}{3} - \frac{3x^2}{2} + 4x\right]_0^3$$

$$= \boxed{\frac{33}{2}}$$

$$= 16.5$$

(10) find lagrange's Interpolating polynomial passing through set of point, 0, 1, 3.

x	0	1	3
y	3	4	12

Use it to find: ① y at $x = 0.5$

② $\frac{dy}{dx}$ at $x = 1$.

Ans: Given:

	x_0	x_1	x_2
x	0	1	3
y	3	4	12
	y_0	y_1	y_2

By lagrange's Intepretation formula,

$$y = P_n(x) = \sum_{i=0}^{n} y_i L_i(x)$$

$$y = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$$y = y_0\left[\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right] + y_1\left[\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right] + y_2\left[\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right].$$

Given:

$x_0 = 0$, $x_1 = 1$, $x_2 = 3$

$y_0 = 3$, $y_1 = 4$, $y_2 = 12.$

$$y = 3\left[\frac{(x-1)(x-3)}{(0-1)(0-3)}\right] + 4\left[\frac{(x-0)(x-3)}{(1-0)(1-3)}\right] + 12\left[\frac{(x-0)(x-1)}{(3-0)(3-1)}\right]$$

$$y = 3\left[\frac{x^2-4x+3}{3}\right] + 4\left[\frac{x^2-3x}{(-2)}\right] + 12\left[\frac{x^2-x}{6}\right]$$

$= x^2-4x+3 + (-2)(x^2-3x) + 2(x^2-x)$

$= x^2-4x+3-2x^2+6x+2x^2-2x$

$\boxed{y = x^2+3}$

① y at $x = 0.5$

$[y]_{x=0.5} =$

② $\frac{dy}{dx}$ at $x=1$.

10/10/25

Finite Difference and Difference operator:-

(1) Forward operator (Δ):→

The difference $\Delta f(x)$ being the difference between forward value of the function and the present value is called forward difference of order first.

And it is defined as $\Delta f(x) = f(x+h) - f(x)$

$\Delta^2 f(x) = \Delta \cdot (\Delta f(x))$

$= \Delta (f(x+h) - f(x))$

$= \Delta f(x+h) - \Delta f(x)$

$= [f(x+2h) - f(x+h)] - [f(x+h) - f(x)]$

$$\boxed{\Delta^2 f(x) = f(x+2h) - 2f(x+h) + f(x)}$$

x	x_0	x_1	x_2	x_3	...	x_n
$f(x)$	$f(x)$	$f(x+h)$	$f(x+2h)$	$f(x+3h)$	...	$f(x+nh)$
y	y_0	y_1	y_2	y_3	...	y_n

or $\Delta y_0 = y_1 - y_0$

$\Delta^2 y_0 = y_2 - 2y_1 + y_0$.

2) Backward operator:- (∇)

The difference $\nabla f(x)$ being the difference between value of the function at the current ~~value~~ point and the preceding point is called backward difference operator.

And it is defined as $\nabla f(x) = f(x) - f(x-h)$.

$$\nabla^2 f(x) = \nabla f(x) - \nabla f(x-h)$$
$$= [f(x) - f(x-h)] - [f(x-h) - f(x-2h)]$$

$$\boxed{\nabla^2 f(x) = f(x) - 2f(x-h) + f(x-2h)}$$

and so on.....

* Newton's forward difference ~~table~~ formula :- [NFDIf]

Newton forward difference formula is given by,

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots$$

where,

$$u = \frac{x - x_0}{h}$$

* forward difference Table:-

	x	y	first diff Δy	second diff $\Delta^2 y$	third diff $\Delta^3 y$	fourth diff $\Delta^4 y$	fifth diff $\Delta^5 y$
1]	x_0	y_0					
			$\Delta y_0 = y_1 - y_0$				
2]	x_0+h	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$			
			$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$		
3]	x_0+2h	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$	
			$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$		$\Delta^5 y_0 = \Delta^4 y_1 - \Delta^4 y_0$
4]	x_0+3h	y_3		$\Delta^2 y_2 = \Delta y_3 - \Delta y_2$		$\Delta^4 y_1 = \Delta^3 y_2 - \Delta^3 y_1$	
			$\Delta y_3 = y_4 - y_3$		$\Delta^3 y_2 = \Delta^2 y_3 - \Delta^2 y_2$		
5]	x_0+4h	y_4		$\Delta^2 y_3 = \Delta y_4 - \Delta y_3$			
			$\Delta y_4 = y_5 - y_4$				
6]	x_0+5h	y_5					

* Newton Backward difference Interpolation formula:-

$$y = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \dots$$

where,

$$u = \frac{x - x_n}{h}$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0	~~$\nabla y_0 = y_1 - y_0$~~				
		$\nabla y_1 = y_1 - y_0$				
x_0+h	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$			
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$		
x_0+2h	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$		$\nabla^4 y_4 = \nabla^3 y_4 - \nabla^3 y_3$	
		$\nabla y_3 = y_3 - y_2$		$\nabla^3 y_4 = \nabla^2 y_4 - \nabla^2 y_3$		
x_0+3h	y_3		$\nabla^2 y_4 = \nabla y_4 - \nabla y_3$			
		$\nabla y_4 = y_4 - y_3$				
x_0+4h	y_4					

Q1] find the value y for value $x = 0.5$ using newton Interpolation formula.

if value is near or start we use forward & if value is after or end use backward

x	0	1	2	3	4
y	1	5	25	100	250

→ By Newton forward difference Interpolation formula.

Here $x = 0.5$, $x_0 = 0$, $h = 1$

$$u = \frac{x - x_0}{h} = \frac{0.5 - 0}{1}$$

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots$$

where, $u = \frac{x - x_0}{h}$.

Given:

$x = 0.5$, $x_0 = 0$, $h = 1$

$u = 0.5 - 0$

$\boxed{u = 0.5}$

By Newton's Difference Interpolation formula,

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots \quad (1)$$

forward difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
	5	4			
1			16		
	25	20		39	
2			55		-19
	100	75		20	
3			75		
	250	150			
4					

from (1)

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{6}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{24}\Delta^4 y_0$$

$$y = 1 + (0.5)(4) + \frac{(0.5)(0.5-1)}{2}(16) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(39) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(-19)$$

$\boxed{y = 4.1796}$

Q2) Using Newton Difference Interpolation Formula find the cubic polynomial which takes the given values.

x	0	1	2	3
y	1	2	1	10

use it to find y at $x = 0.5$

→

x	0	1	2	3
y	1	2	1	10

here $x = x$, $x_0 = 0$, $h = 1$

$u = \frac{x - x_0}{h} = \frac{x - 0}{1}$

$\boxed{u = x}$

By newton's forward difference Interpolation formula.

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \quad -(1)$$

forward difference Table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

from (1),

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{6}\Delta^3 y_0$$

$$y = 1 + x(1) + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12)$$

$$= 1 + x - (x^2 - x) + 2(x(x^2 - 3x + 2))$$

$= 1 + x - x^2 + x + 2x^3 - 6x^2 + 4x$

$\boxed{y = 1 + 6x - 7x^2 + 2x^3}$

At $x = 0.5$

$y = 1 + 6(0.5) - 7(0.5)^2 + 2(0.5)^3$

$\boxed{y = 2.5}$

Q) The distance travel by a point p in xy plane in a mechanism is as shown in the table below estimate the distance travel, velocity acceleration at point P when $x = 4.5$

x	1	2	3	4	5
y	14	30	62	116	198

Here, $x = x$, $x_0 = 5$, $h = 1$

$u = \frac{x - x_n}{h} = \frac{x-5}{1}$

$\boxed{u = x - 5}$

By Newton's Backward Diff Interpolation formula

$y = y_n + u \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n$ ... ①

Backward Difference Table.

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	14				
		16			
2	30		16		
		32		6	
3	62		22		
		54		6	
4	116		28	$\nabla^3 y_n$	$\nabla^4 y_n$
		82	$\nabla^2 y_n$		
5	198 y_n	∇y_n			

from ①

$y = y_n + u \nabla y_n + \frac{u(u+1)}{2} \nabla^2 y_n + \frac{u(u+1)(u+2)}{6} \nabla^3 y_n$

$y = 198 + (x-5)82 + \frac{(x-5)(x-4)}{2}(28) + \frac{(x-5)(x-4)(x-3)}{6} \times 6$

$= 198 - 410 + 82x + 14(x^2 - 9x + 20) + (x^2 - 9x + 20)(x-3)$

$= -212 - 82x + 14x^2 - 126x + 280 + x^3 - 9x^2 + 20x$

$- 3x^2 + 27x - 60$

$\boxed{y = 8 + 3x + 2x^2 + x^3}$

(i) Distance Travel at $x = 4.5$

$y = 8 + 3(4.5) + 2(4.5)^2 + (4.5)^3$

$\boxed{y = 15.125}$

(ii) Velocity $(v) = \frac{dy}{dx} = 3 + 4x + 3x^2$

v at 4.5

$v = 3 + 4(4.5) + 3(4.5)^2$

$\boxed{v = 81.75}$

(iii) $ACC = \frac{dv}{dt} = 4 + 6x$.

a at 4.5

$\boxed{a = 31}$

Q) Using NBF find the value of y when $x = 7$

x	0	2	4	6	8
y	5	29	125	341	725

here $x = 7$, $x_n = 8$, $h = 2$

$u = \frac{x - x_n}{h} = \frac{7-8}{2} = \frac{-1}{2}$

$\boxed{u = 0.5}$

By Newton's Backward diff Interpolation formula

$y = y_n + u y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n + \ldots$

Backward diff table,

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	5				
		24			
2	29		72		
		96		48	
4	125		120		0
		216		48	
6	341		168		
		384			
8	725				

from ①

$y = y_n + u \nabla y_n + \frac{u(u+1)}{2} \nabla^2 y + \nabla^3 y_n \frac{(u+1)(u+2)}{6} \nabla^3 y_n$

$= 725 + (0.5)(384) + \frac{(-0.5)(0.5)}{2} \times 168 + \frac{(0.5)(0.5)(1.5)}{6} \times 48$

$= \boxed{y = 509}$

(Q) Using Newton Interpolation Formula. find the value of y when $x = 5$

x	4	6	8	10
y	1	3	8	16

$\rightarrow$ $x = 5$, $x_0 = 4$, $h = 2$

$u = \frac{x - x_0}{h} = \frac{5-4}{2} = \frac{1}{2}$.

$\boxed{u = 0.5}$

By Newtons forward diff Interpolation formula

$y = y_0 + u \Delta y_0 + \frac{u(u+1)}{2!} \Delta^2 y_0 + \frac{u(u+1)(u+2)}{3!} \Delta^3 y_0 + \ldots$ ①

forward diff table,

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
4	1				
		2			
6	3		3		
		5		2	
8	8		5		
		8			
10	16.				

from ①

$y = y_0 + u \nabla y_0 + \frac{u(u+1)}{2} \nabla^2 y_0 + \frac{u(u+1)(u+2)}{6} \nabla^3 y_n$

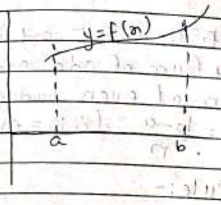
$y =$

1/4/25 10 marks

3) Numerical Integration:

- A process of evaluating a definite integral from a set of tabulated value from the $f(x)$ is called numerical Integration.

$I = \int_a^b f(x)\,dx.$



10. Trapezoidal Rule:→

The definite Integral $I = \int f(x)\,dx$ of a Continous function $f(x)$ on the Interval $[a,b]$ can be approximated using n-sub-interval

As follows,

It is define as:-

1) $\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\ldots+y_{n-1})\right]$

$= \frac{h}{2}\left[(\text{sum of first \& last ordinates})+ 2(\text{Sum of remaining ordinates})\right]$

where, $h = \frac{b-a}{n}$ & $x_i = x_0 + ih$.

10. Simpson's $\left(\frac{1}{3}\right)^{rd}$ Rule:-

Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule is defined by,

∴ for all three rule are applicable.

where $\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+\dots y_{n-1})+2(y_2+y_4+\dots\dots+y_{n-2})\right]$

$= \frac{h}{3}\left[(\text{Sum of first \& last ordinate})+4(\text{Sum of odd ordinates})+2(\text{Sum of even ordinates})\right]$

∴ where, $h = \frac{b-a}{n}$ (∴ n = even)

10 ★ Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule:-

where: $h = \frac{b-a}{n}$ (∴ ~~n = even~~)

Simpson's $\left(\frac{3}{8}\right)^{th}$ rule is defined as

$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_n)+2(y_3+y_6+y_9+\dots)+3(y_1+y_2+y_4+y_5+\dots\dots)\right]$

$= \frac{3h}{8}\left[(\text{Sum of first \& last ordinate})+2(\text{Sum of multiple of three ordinate})+3(\text{Sum of remaining ordinates})\right]$

∴ where, $h = \frac{b-a}{n}$ (n = multiple of 3)

Note:-

① There is no restriction for the Number of Interval in the trapezoidal rule.

② In simpson's $\left(\frac{1}{3}\right)^{rd}$ rule no. of Sub interval must be even.

③ In Simpson's $\left(\frac{3}{8}\right)^{th}$ rule the no. of sub. interval must be multiple of 3.

• Example:
Use Trapezoidal Rule to estimate the value of: $\int_0^1 x^3 dx$. Considering 5 sub Interval.

Ans: here,
$y = x^3$ and $n = 5$

No. of Subinterval = (Total number of ordinate) − 1

∴ $h = \frac{b-a}{n} = \frac{1-0}{5} = \frac{1}{5}$

Total number of ordinate = 1 + No. of subinterval.

∴ $\boxed{h = \frac{1}{5}}$

x	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5} = 1.$
y	0	0.008	0.064	0.216	0.512	1.
y_0	y_0	y_1	y_2	y_3	y_4	y_5

By Trapezoidal Rule:

$\int_0^1 y\,dx = \frac{h}{2}\left[(y_0+y_5)+2(y_1+y_2+y_3+y_4)\right]$

$= \frac{\left(\frac{1}{5}\right)}{2}\left[(0+1)+2(0.008+0.064+0.216+0.512)\right]$

$= \frac{1}{10}\left[1+1.6\right]$

$\boxed{\int_0^1 x^3 dx = 0.26.}$

② find the Area bounded by the curve $f(x) = \frac{1}{1+x^2}$ for $x = 0$, $x = 4$ by taking 9 ordinate. Using Trapezoidal rule.

→ Given:-

$f(x) = \frac{1}{1+x^2}$

no. of ordinate = 9.

$\therefore$ no. of subinterval = 9 − 1 = 8

$\therefore n = 8$

$h = \frac{b-a}{n}$

$= \frac{4-0}{8} = \frac{4}{8} = \frac{1}{2}$

$\boxed{h = \frac{1}{2}}$ //

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
y	1	0.8	0.5	0.3076	0.2	0.1379	0.1	0.0754	0.0588
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

By Trapezoidal Rule:

$$\int_0^4 y\,dx = \frac{h}{2}\left[(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$$= \frac{\frac{1}{2}}{2}\left[(1 + 0.0588) + 2(0.8 + 0.5 + 0.3076 + 0.2 + 0.1379 + 0.1 + 0.0752)\right]$$

$$= \frac{1}{4}\left[1.0588 + 4.2428\right]$$

$$\boxed{\int_0^4 \frac{1}{1+x^2}\,dx = 1.3254}$$ //

1.3850

③ Use Trapezoidal Rule to estimate the value of $\int_0^1 xe^{x^2}\,dx$ by taking $h = 0.1$.

→ Given:-

$f(x) = \int_0^1 xe^{x^2}\,dx$, $x = 0$, $x = 1$

~~no. of ordinate~~ $h = 0.1$

$n = \frac{b-a}{h}$

$n = \frac{1-0}{0.1} = 10.$

$\boxed{n = 10}$ //

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
y	0	0.101	0.2082	0.3283	0.4694	0.642	0.8599	1.142
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7

x	0.8	0.9	1
y	1.51718	2.0231	2.718
	y_8	y_9	y_{10}

7/4/25

(Q) Evaluate:-

$\int_0^{0.8} [\log_e(1+x) + \sin(2x)]\,dx$ where x is in radian. Use Simpson $\frac{1}{3}$rd rule divide the entire interval into 8th part.

→ let $I = \int_0^{0.8} [\log e(1+x) + \sin(2x)]\,dx$.

here $y = f(x) = \log e(1+x) + \sin(2x)$

$n = 8$.

$h = \frac{b-a}{n} = \frac{0.8-0}{8}$

$\boxed{h = 0.1}$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
y	0	0.2939	0.5717	0.8270	1.0538	1.24693	1.40204	1.51607	1.58736
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

By Simpson $\frac{1}{3}$rd rule,

$$\int_0^{0.8} y\,dx = \frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\right]$$

$$= \frac{0.1}{3}\left[(0+1.5873)+4(0.2939+0.8270+1.2493+1.5160)+2(0.5717+1.0538+1.40204)\right]$$

$$= 0.0333\left[(1.5873)+15.53532+6.05508\right]$$

$$= 0.333(23.1777) = \boxed{0.7725}$$

Q) Use Simpson's 1/3rd rule to obtained $\int_0^{\pi/2} \frac{\sin x}{x}\,dx$ dividing the interval into 4 part.

Ans let $I = \int_0^{\pi/2} \frac{\sin x}{x}\,dx$

here, $y = f(x) = \frac{\sin x}{x}$

$n = 4$.

$h = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$

$\boxed{h = \frac{\pi}{8}}$.

x	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$
y	1	0.9744	0.9003	0.7842	0.6366
	y_0	y_1	y_2	y_3	y_4

By Simpson's $\frac{1}{3}$rd rule,

$$\int_0^{\pi/2} y\,dx = \frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3+\cancel{y_5+y_7})+2(y_2+y_4+y_6)\right]$$

$$= \frac{\frac{\pi}{8}}{3}\left[(1+0.6366)+4(0.9744+0.7842)+2(0.9003+0.6366)\right]$$

$$= 1.17\left[1.6366 + 10.1082\right]$$

$$= 1.17\,(11.7448)$$

$$= \boxed{1.3707}$$

Q) Evaluate: $\int_0^{\pi} \frac{\sin^2 x}{5+4\cos x}\,dx$ by Simpson $\frac{3}{8}$th rule

Taking $h = \pi/6$

→ let $I = \int_0^{\pi} \frac{\sin^2 x}{5+4\cos x}$

here, $y = f(x) = \frac{\sin^2 x}{5+4\cos x}$

$h = \frac{\pi}{6}$

$n = \frac{b-a}{h} = \frac{\pi - 0}{\pi/6} = \boxed{6}$.

$\boxed{n = 6}$

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$	π
y	0	0.0295	0.1071	0.2	0.25	0.1627	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\int_0^{\pi} y\,dx = \frac{3h}{8}\left[(0+0) + 2(0.2) + 3(0.0295 + 0.1071 + 0.25 + 0.1627)\right]$

$= \frac{3 \frac{\pi}{6}}{8}\left[2.0479\right]$

$= 0.1963 \left[2.0479\right]$

$= 0.4020$

Q) Evaluate:- $\int_0^3 \frac{dx}{1+x}$ with 7 ordinate by using Simpson $\frac{3}{8}$th rule.

→ let:- $I = \int_0^3 \frac{dx}{1+x}$

$y = f(x) = \frac{1}{1+x}$.

$n = 7-1 = 6$.

$h = \frac{b-a}{n} = \frac{3-0}{6} = \frac{3}{6} = \frac{1}{2}$

x	0	1/2	1	3/2	2	5/2	3
y	1	0.666	0.5	0.4	0.3333	0.2857	0.25
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\int_0^3 y\,dx = \frac{3h}{8}\left[(1+0.25) + 2(0.4) + 3(0.6+0.5+0.3333+0.2857)\right]$

$= \frac{3 \times \frac{1}{2}}{8}\left[(1.25) + 0.8 + 5.157\right]$

$= 0.1875\left[7.207\right]$

$= 1.3513$

$\int_0^3 y\,dx = 1.3888$

classmate

Date ____
Page ____

(1) Trapezoidal rule
(2) Sim 1/3rd rule

Q) Evaluate:

$\int_0^6 xe^x dx$, $h = 1.0$ using Simpson's 3/8th rule

→ Given:

let $f(x) = \int_0^6 xe^x dx$.

$h = 1.0$

$n = \frac{b-a}{h}$

$n = \frac{6-0}{1.0}$

$\boxed{n = 6}$

x	0	1	2	3	4	5	6
y	0	2.7182	14.778	60.256	216.392	742.065	2420.5727
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(1) $\int_0^6 y dx = \frac{h}{2} \left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots y_n) \right.$

$= \frac{1.0}{2} \left[(0 + 2420.57) + 2(2.7182 + 14.778 + 60.256 + 218.392 + 742.065) \right]$

$= 0.5 \left[4496.9884 \right)$

$\boxed{\int_0^6 y dx = 2248.4942.}$

(2) $\int_0^6 y dx = \frac{h}{3} \left[y_0 + y_n + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4 + y_6) \right)$

$= \frac{1.0}{3} \left[(0 + 2420.57) + 4(2.7182 + 60.256 + 742.065) + 2(14.778 + 218.392 + 2420.5727) \right)$

$= 0.333 \left[6107.0668 \right)$

$= \boxed{2033.653}$

(u) Numerical Solution of ordinary differential Equation.

① Modified Euler's method:

$$\frac{dy}{dx} = f(x, y) \qquad y = y_0 \text{ at } x = x_0 \text{ or}$$

$$y(x_0) = y_0.$$

To start, to use Euler's formula, to find the value of y at $x = x_1$.

$\therefore$ ~~y1(0)~~ $y_1^{(0)} = y_1 \approx y_0 + h f(x_0, y_0)$
where $y_1^{(0)}$ is zeroth approximation,

* To find more correct approximation at $x = x_1$ we use Euler's modified itteration formula,

$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(n)})\right] \quad n = 0, 1, 2, 3 \ldots$$

* Example:-
Use modified Eulers method to solve $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ to Calculate $y(0.1)$ taking $h = 0.1$.

→ Given:

$$\frac{dy}{dx} = x^2 + y, \quad y(0) = 1 \qquad h = 0.1$$

$$\therefore f(x, y) = x^2 + y \qquad y(x_0) = y_0$$

$x_0 = 0$, $y_0 = 1$

here $x_1 = x_0 + h$
$= 0 + 0.1$
$\boxed{x_1 = 0.1}$

$f(x, y) = x^2 + y$
$f(x_0, y_0) = x_0^2 + y_0$
$= 0^2 + 1$
$\boxed{f(x_0, y_0) = 1}$

To find y_1 at $x_1 = 0.1$.
By Euler's formula,

$$y_1(0) = y_1 \approx y_0 + h f(x_0, y_0)$$
$$= 1 + 0.1 [0^2 + 1)$$
$$y_1^{(0)} = 1 + 0.1(1)$$
$$\boxed{y_1^{(0)} = 1.1}$$

$\therefore$ By modified Euler's formula,

$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(n)})\right] \quad n = 0, 1, 2, 3$$

put $n = 0$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$1.05 + f(1.11)$

$$= 1 + \frac{0.1}{2}\left[(0^2 + 1) + f[(0.1)^2 + 1.1)\right]$$

$$\boxed{y_1^{(1)} = 1.1055}$$

put $n = 1$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$
$$= 1 + \frac{0.1}{2}\left[(0^2 + 1)(0.1)^2 + 1.1055\right]$$

$$\boxed{y_1^{(2)} = 1.1057}$$

put $n = 2$.

$$\boxed{y_1^{(3)} = 1.105781}$$

(Q) Solve the Equation $\frac{dy}{dx} = 1+xy$, $y(0)=1$ to find y at $x=0.1$ using modified Euler's method taking $h=0.1$ correct upto 3 decimal places.

Ans: Given:

$\frac{dy}{dx} = 1+xy$, $y(0)=1$

$\therefore f(x,y) = 1+xy$

$x_0 = 0$, $y_0 = 1$, $h = 0.1$

$x_1 = x_0 + h$

$= 0+0.1$

$\boxed{x_1 = 0.1}$

To find y at $x_1 = 0.1$

By Euler's formula,

$y_1(0) = y_1 \approx y_0 + hf(x_0, y_0)$

$= 1+0.1[1+(0)(1)]$

$y_1^{(0)} = 1+0.1(1)$

$\boxed{y_1^{(0)} = 1.1}$

By Euler's modified Euler's formula

$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(n)})\right]$

$n = 0, 1, 2, 3.$

put $n=0$

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(0)})\right]$

$y_1^{(1)} = 1 + \frac{0.1}{2}\left[(1+(0)(1)) + (1+(0.1)(1.1))\right]$

$\boxed{y_1^{(1)} = 1.1055}$

put $n=1$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(1)})\right]$

$y_1^{(2)} = 1 + \frac{0.1}{2}\left[(1+(0)(1)) + (1+(0.1)(1.1055))\right]$

$y_1^{(2)} = 1.105527.$

put $n=2$.

$y_1^{(3)} = 1.1055$

(Q) $\frac{dy}{dx} = \log(x+y)$, $y(1)=2$ — correct upto 3 decimal places

$f(x,y) = \log(x+y)$

$x_0 = 0$, $y_0 = 1$, $h = 0.2$

here $x_1 = x_0 + h$

$= 1+0.2$

$\boxed{x_1 = 1.2}$

To find y_1 at $x_1 = 1.2$

By Eulers formula,

$y_1(0) = y_1 \approx y_0 + hF(x_0, y_0)$

$= 2 + 0.2[\log(1+2)]$

$y_1^{(0)} = 2 + 0.2(1.0986)$

$\boxed{y_1^{(0)} = 2.2197}$

$\therefore$ By modified Euler's formula,

$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(n)})\right]$

put $n=0$

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(n)})\right]$

$y_1^{(1)} = 2 + \frac{0.2}{2}\left[\log(1+2) + \log(1.2+2.2197)\right]$

$\boxed{y_1 = 2.2328}$

put $n=1$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(1)})\right]$

$= 2 + \frac{0.2}{2}\left[\log(1+2) + \log(1.2 + 2.2328)\right]$

$\boxed{y_1^{(2)} = 2.2331}$

put $n=2$.

$y_1^{(3)} = 2 + \frac{0.2}{2}\left[\log(1+2) + \log(1.2 + 2.2331)\right]$

$\therefore \boxed{y_1^{(3)} = 2.2332} \Rightarrow$ Ans.

Q) Solve eqn : $\frac{dy}{dx} = xy \quad y(0) = 1.$

$f(x,y) = \log(x+y)$

$x_0 = 0, \; y_0 = 1, \; h = 0.1$ Correct upto 3 decimal places

Here, $x_1 = x_0 + h$

$= 0 + 0.1$

$\boxed{x_1 = 0.1}$

To find y_1 at $x_1 = 0.1$.

$y_1^{(0)} = y_1 \approx y_0 + hf(x_0, y_0)$

$= 1 + 0.1\,[(0)(1)]$

$y_1^{(0)} = 1 + (0.1)(0)$

$\boxed{y_1^{(0)} = 1}$

put $n=0$.

$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(0)})\right].$

$y_1^{(1)} = 1 + \frac{0.1}{2}\left[(0)(1) + (0.1)(1)\right]$

$= 1.005$//.

put $n=1$

$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0,y_0) + f(x_1, y_1^{(1)})\right]$

$= 1 + \frac{0.1}{2}\left[(0)(1) + (0.1)(1.005)\right]$

$\boxed{y_1(2) = 1.0050}$//

put $n=2$

$\boxed{y_1(3) = 1.0050}$

* Runge - Kutta method of fourth order:-

Is most commonly used; working rule for finding the increment of y corresponding to h of x by Runge-Kutta method from, $\frac{dy}{dx} = F(x,y)$, $y(x_0) = y_0$ as follows,

$k_1 = hF(x_0, y_0)$

$k_2 = hF\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$k_4 = hf(x_0 + h, y_0 + k_3)$

finally $k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$y_1 = y_0 + k.$

Q) Use Runge-Kutta fourth order method to find the value of y at $x = 1.2$ taking $h = 0.2$ given that $\frac{dy}{dx} = \log(x+y)$, $y(1) = 2.$

→ Given:

$\frac{dy}{dx} = \log(x+y) \quad y(1) = 2$

here $f(x,y) = \log(x,y)$

$x_0 = 1, \ y_0 = 2, \ h = 0.2$

$x_1 = x_0 + h$

$= 1 + 0.2$

$\boxed{x_1 = 1.2}$

$k_1 = hf(x_0, y_0)$

$= (0.2)\log(1+2)$

$\boxed{k_1 = 0.2197}$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.2)\ \cancel{\log f}\left(1 + \frac{0.2}{2}, 2 + \frac{0.2197}{2}\right)$

$= (0.2) f(1.1 + 2.1098)$

$\cancel{k_2 = 2.3324}$ $\boxed{k_2 = 0.2332}$

$k_3 = 0.2 \log\left(1.1 + \frac{0.2}{2} + 2.1166\right)$

$\boxed{k_3 = 0.23366}$

$k_4 = 0.2\log(1.2 + 2.23366)$

$\boxed{k_4 = 0.2467}$

$\therefore k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= \frac{1}{6}\left(0.2197 + 2(0.2332) + 2(0.23366) + (0.2467)\right)$

Q) Use Runge-Kutta method of fourth order:- to find the value of y where $x = 0.2$, Given that $\frac{dy}{dx} = 1+y^2$, $y(0) = 1$.

→ Given that

$\frac{dy}{dx} = 1+y^2, \ y(0) = 1 \quad h = 0.2$

$f(x,y) = 1+y^2$

$x_0 = 0 \quad y_0 = 1, \ h = 0.2$

Now,

$x_1 = x_0 + h$

$= 0 + 0.2$

$\boxed{x_1 = 0.2}$

$k_1 = hf(x_0, y_0)$.

$= 0.2 f(0, 1)$

$= 0.2(1+1)$

$\boxed{k_1 = 0.4}$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$

$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0.4}{2}\right)$

$= 0.2 f\left(0.1 + (1.2)^2\right)$

$= 0.2 f\ \cancel{(1.3)}\ (1 + (1.2)^2)$

$\cancel{k_2 = 0.26}$, $\boxed{k_2 = 0.488}$

$k_3 = 0.2 f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.26}{2}\right)$

$= 0.2(0.1 + 1.13)$

$= 0.2(1.23)$

$\boxed{k_3 = \cancel{0.246}}$ 0.05095.

$k_4 = hf(x_0+h, y_0+k_3)$

$k_4 = 0.2(0+0.2, 1+0.5095)$

$= 0.2(0.2+1.5095)$

$\boxed{k_4 = 0.6557}$

$= \frac{1}{6}(k_1+2k_2+2k_3+k_4)$

$= \frac{1}{6}(0.4+2(0.488)+2(0.5095)+0.6557)$

$= \frac{1}{6}(3.0507)$

$= \boxed{0.50845}$

$\therefore$ Required approximate value of

$y_1 = y_0+k$

$= 1+0.5084$

$\boxed{y_1 = 1.5084}$ //.

Unit-IV] Vector Calculus.

① D.D.

② Stoke's theorem

③ Green's theorem | solenoid.

Unit-3 ① polar, Area, Cartesian coordinate.

② Volume

* Unit - II *

Beta Integral

Gamma, DUIS, reduction in integration.

— Unit - I *

* Newton law of Cooling.
* Electric Circuit (LR & CR)
* Non-Exact → Rule → 1 & 2.

edu GAMES

LEARN WHILE YOU PLAY

Improve Your Logical Reasoning With This Activity.

Four people of different nationalities live on the same side of a street in four houses each of different colour. Each person has a different favourite drink.

Following are the additional information known:

The Englishman lives in the red house.

The Italian drinks tea.

The Norwegian lives in the first house on the left.

In the second house from the right they drink milk.

The Norwegian lives adjacent to the blue house.

The Spaniard drinks fruit juice.

Tea is drunk in the blue house.

The White House is to the right of the red house.

~~Any two based on Unit-IV~~.

Unit-V: Numerical methods

① Sol. of algebraic & Trance equation.
→ Bisection method
→ N-R method

② Interpolation (10)
→ lagrange's
→ NFDIF
→ NBDIF

Numerical Integration (10)
- Trapezoidal
- Simpson's $(1/3)^{rd}$
- Simpson's $(\frac{3}{8})^{th}$.

Numerical Solution of ODE (5)
- modified Eulers
- Runge Kutta method

Question: Milk is drunk by the

(1) Norwegian, (2) Englishman, (3) Italian, (4) None of these

Question: The Norwegian drinks

(1) Milk, (2) Cocoa, (3) Tea, (4) Fruit Juice

Question: The colour of the Norwegian's house is

(1) Yellow, (2) White, (3) Blue, (4) Red

Question: Which of the following is not true:

(1) Milk is drunk in the red house.
(2) The Italian lives in the blue house.
(3) The Spaniard lives in a corner house.
(4) The Italian lives next to Spaniard.